Name: ______________________________

Instructor: __________________________

Math 10170, Practice Exam I
February 22, 2016

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 13 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______________
8.	______________
9.	______________
10.	______________
11.	______________
12.	______________
Total	______________

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Multiple Choice

1.(6 pts.) Twenty members of a lacrosse squad are trying to decide what type of food to order after their game. Each member of the group has listed their preferences and the results are shown in the table below. The group will use the Borda Method (average rank) to decide which type of food to offer.

# of Voters = 20

	5	**7**	**8**	Ave.
Sushi	1	2	3	$\frac{5+14+24}{20} = \frac{43}{20}$
Hamburgers	2	1	1	$\frac{10+7+8}{20} = \frac{25^*}{20}$
Hot Dogs	3	3	2	$\frac{15+21+16}{20} = \frac{52}{20}$
Sandwiches	4	4	4	$\frac{20+28+32}{20} = \frac{80}{20}$
Pizza	5	5	5	$\frac{25+35+40}{20} = \frac{100}{20}$

The winner using the Borda Method is: Hamburgers.

(a) Sushi (b) Hot Dogs (c) Sandwiches

(d) Pizza (e) Hamburgers

2.(6 pts.) Consider the following matrices:

$$A = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Which of the following matrices is equal to $BA - C$?

$$B \cdot A - C = \begin{pmatrix} 0 & 2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}.$$

(a) $\begin{pmatrix} 11 \\ -2 \end{pmatrix}$. (b) $\begin{pmatrix} 3 \\ 11 \end{pmatrix}$. (c) $\begin{pmatrix} 13 \\ 2 \end{pmatrix}$. (d) $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$. (e) $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$.

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3.(6 pts.) There are four dorms competing in the Notre Dame Spring Olympics. There are 10 events golf, rugby, 100 meter race, 5K race, darts, snooker, karate, boxing, soccer and Irish dancing. Each dorm has participants in each event. The table below shows the results for 2013 for the ten events (with first in an event denoted by 1).

Number of Events

	1	**2**	**1**	**1**	**2**	**3**
Morrisey Hall	1	4	2	3	2	4
Walsh Hall	2	3	3	1	1	3
Lyons Hall	4	1	1	4	4	1
O'Neill Hall	3	2	4	2	3	2

Each year "The Olympic Cup" is awarded one of the dorms based on their overall performance. If a Condorcet winner exists, the Olympic cup is awarded to that dorm, otherwise a Condorcet completion process is used to decide the winner. Which of the following is true?

$$L(6)\ v\ M(4) \to L,$$

$$L(6)\ v\ W(4) \to L,$$

$$L(6)\ v\ O'N(4) \to L$$

Lyons is the Condorcet winner.

(a) Walsh is the Condorcet winner

(b) Lyons is the Condorcet winner

(c) O'Neill is the Condorcet winner

(d) Morrisey is the Condorcet winner

(e) There is no Condorcet winner

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4.(6 pts.) An athlete is planning a diet for a twelve week training plan. She has a prescribed balance of carbohydrates, protein and fat for each meal. The athlete will three foods for tomorrow's breakfast.

One ounce of Food 1 has 30% of the required carbohydrates, 20% of the required protein and 0.1% of the required fat.

One ounce of Food 2 has 10% of the required carbohydrates, 40% of the required protein and 0% of the required fat.

One ounce of Food 3 has 0% of the required carbohydrates, 0% of the required protein and 50% of the required fat.

Let x denote the number of ounces of food 1 she will include, let y denote the number of ounces of food 2 she will include and let z denote the number of ounces of food 3 she will include. if she aims to have exactly 100% of the prescribed quantities of carbohydrates, protein and fat in her breakfast, which system of equations must she solve:

We want 100% of the required protein, the amount of protein we get from x ounces of Food 1 is 20x% of our required daily intake. The amount of protein we get from y ounces of Food 2 is 40y% of our required daily intake and the amount of protein we get from z ounces of Food 3 is $0z = 0\%$ of our required daily intake. We want our protein intake to add to 100% of our required intake, thus we want

$$20x + 40y + 0z = 100.$$

Likewise we get equations from setting our Carbohydrate intake and our fat intake to 100% of the required amount:

$$\text{Carb.} \quad 30x + 10y + 0z = 100$$

$$\text{Fat} \quad 0.1x + 0y + 50z = 100.$$

We see that (a) below matches our system of equations.

(a)
$$\begin{aligned} 30x + 10y + 0z &= 100 \\ 20x + 40y + 0z &= 100 \\ 0.1x + 0y + 50z &= 100 \end{aligned}$$

(b)
$$\begin{aligned} 30x + 20y + 0.1z &= 100 \\ 10x + 40y + 0z &= 100 \\ 0x + 0y + 50z &= 100 \end{aligned}$$

(c)
$$\begin{aligned} 3x + 1y + 0z &= 100 \\ 2x + 4y + 0z &= 100 \\ 0.01x + 0y + 5z &= 100 \end{aligned}$$

(d)
$$\begin{aligned} 30x + 10y + 0z &= 100 \\ 20x + 50y + 0z &= 100 \\ 0.1x + 0y + 40z &= 100 \end{aligned}$$

(e)
$$\begin{aligned} 30x + 0y + 10z &= 100 \\ 20x + 0y + 40z &= 100 \\ 0.1x + 50y + 0z &= 100 \end{aligned}$$

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5.(6 pts.) The following table shows the results of a round robin in progress (up to Feb. 24 2014 the Six Nations Championship in Rugby).

	Ireland	England	Wales	Scotland	France	Italy	P-D
Ireland		10-13	26-3	28-6			42
England	13-10			20-0	24-26		21
Wales	3-26				27-6	23-15	6
Scotland	6-28	0-20				21-20	-41
France		26-24	6-27			30-10	1
Italy			15-23	20-21	10-30		-29

Which of the following matrix equations must be solved in order to find the Massey Ratings (keeping the same ordering of the teams as above)?

Each team has played 3 matches so far, therefore (c) is the correct answer below.

(a)
$$\begin{pmatrix} 5 & -1 & -1 & -1 & 0 & 0 \\ -1 & 5 & 0 & -1 & -1 & 0 \\ -1 & 0 & 5 & 0 & -1 & -1 \\ -1 & -1 & 0 & 5 & 0 & -1 \\ 0 & -1 & -1 & 0 & 5 & -1 \\ 0 & 0 & -1 & -1 & -1 & 5 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \\ 3/2 \\ 1/2 \\ 3/2 \\ -1/2 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 5 & -1 & -1 & -1 & 0 & 0 \\ -1 & 5 & 0 & -1 & -1 & 0 \\ -1 & 0 & 5 & 0 & -1 & -1 \\ -1 & -1 & 0 & 5 & 0 & -1 \\ 0 & -1 & -1 & 0 & 5 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \\ 3/2 \\ 1/2 \\ 3/2 \\ 0 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 & 0 \\ -1 & 0 & 3 & 0 & -1 & -1 \\ -1 & -1 & 0 & 3 & 0 & -1 \\ 0 & -1 & -1 & 0 & 3 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 42 \\ 21 \\ 6 \\ -41 \\ 1 \\ 0 \end{pmatrix}$$

(d)
$$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 & 0 \\ -1 & 0 & 3 & 0 & -1 & -1 \\ -1 & -1 & 0 & 3 & 0 & -1 \\ 0 & -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & -1 & -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 42 \\ 21 \\ 6 \\ -41 \\ 1 \\ -29 \end{pmatrix}$$

(e) None of the above

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6.(6 pts.) An experiment consists of flipping a coin until a tail appears. As soon as a tail appears, the experimenter stops and records the sequence of heads and tails. What is the probability that the outcome of this experiment is $HHHHHHT$?

The probability that this configuration would happen in 7 flips of a coin is $\frac{1}{2^7} = \frac{1}{128}$.

(a) $\frac{1}{12}$ (b) $\frac{1}{64}$ (c) $\frac{1}{4}$ (d) $\frac{1}{128}$ (e) 0

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7.(6 pts.) If a basketball player attempts 250 shots in a row with a 50% chance of making a basket on each shot, what is the longest run of baskets you would expect in the sequence of outcomes, based on probability.

In a sequence of K shots by the player, we would expect the longest run to have length approximately

$$\frac{\ln\left(\frac{K}{2}\right)}{\ln 2}.$$

Thus in a sequence of 250 shots, we expect a longest run of length approximately

$$\frac{\ln\left(\frac{250}{2}\right)}{\ln 2} = \frac{\ln(125)}{\ln 2} = 6.96 \approx 7.$$

(a) 8 (b) 6 (c) 7 (d) 4 (e) 10

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Partial Credit

You must show your work on the partial credit problems to receive credit!

8.(10 pts.) There are five dorms competing in the Notre Dame Spring Olympics. There are 7 events; Sumo Wrestling, Polo, Jai Alai, Archery, Raft Racing, Baseball and Basketball. Each dorm has participants in each event. The table below shows the results for 2013 for the six events (with first in an event denoted by 1). It also shows the Borda count for each dorm.

Number of Events

	1	**2**	**1**	**2**	**1**	**Ave**
Dillon Hall	4	3	5	3	4	25/7*
Knott Hall	2	1	2	5	3	19/7
Lewis Hall	3	2	4	4	5	24/7*
McGlinn Hall	5	4	3	1	2	20/7
Carroll Hall	1	5	1	2	1	17/7
					Ave	21/7

Each year "The Olympic Cup" is awarded one of the dorms based on their overall performance. If a Condorcet winner exists, the Olympic cup is awarded to that dorm. If not Nanson's method is used to complete the process.

(a) Show that there is no Condorcet winner for the above tournament.

Every team is beaten in a head to head comparison at least once:

	DvK	LvK	McGvK	KvC	CvMcG
	2v5	3v4	3v4	2v5	3v4
Winner	K	K	K	C	McG

(b) Apply **Nanson's method** to find the winner of the olympic cup?

Number of Events

	1	**2**	**1**	**2**	**1**	**Ave**
Knott Hall	2	1	2	3	3	15/7*
McGlinn Hall	3	2	3	1	2	14/7
Carroll Hall	1	3	1	2	1	13/7
					Ave	14/7

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Number of Events

	1	**2**	**1**	**2**	**1**	**Ave**
McGlinn Hall	2	1	2	1	2	10/7
Carroll Hall	1	2	1	2	1	11/7
					Ave	14/7

McGlinn Wins

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9.(10 pts.)

$$\begin{aligned} x + 5y + z &= 8 \\ 2x + 15y + z &= 20 \\ x + y + 2z &= 5 \end{aligned}$$

(a) Write the above system of equations as a matrix equation.

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 15 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 20 \\ 5 \end{bmatrix}$$

(b) Write the following system of equations as a matrix equation $AX = B$.

$$\begin{aligned} x + 2y &= 3 \\ x + y &= 2 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

(c) Solve the system in part (b) by finding the matrix A^{-1} and multiplying the equation by A^{-1}. (show your work for credit).

Det(A) $= 1 \cdot 1 - 1 \cdot 2 = -1$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$A^{-1} = \underline{\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}}$ Solution: $x = \underline{1}$, $y = \underline{1}$.

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10.(10 pts.) (A) Which of the following describes the property of independence from irrelevant alternatives in a voting system?

(a) There are no restrictions placed on the ranking of the candidates a voter may choose. Universal Domain

(b) If all voters prefer candidate A to candidate B, then the group choice should not prefer candidate B to candidate A. Pareto Optimality

(c) No one individual voter preference totally determines the group choice. Non-Dictatorship

(d) If a group of voters choose candidate A over candidate B,
then the addition or subtraction of other candidates should not change the group choice to B. Independence from Irrelevant Alternatives

(e) If choice A is the winner of an election and, in a reelection, the only changes in the ballots are changes that only favor A, then A should remain the winner of the election. Monotonicity.

(B) If there are 15 teams in a round robin tournament, how many games must be played?

$$\frac{15 \times 14}{2} = 105.$$

(C) If an experiment has 35equally likely outcomes, what probability should be assigned to each?

$$\frac{1}{35}$$

(D) if an experiment consists of flipping a coin 10 times in a row, what is the probability that all of the outcomes will be tails?

$$\frac{1}{2^{10}} = \frac{1}{1024}.$$

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11.(10 pts.) This problem appears as Problem 1 on the take home part of the exam. You may use this page for rough work.

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12.(18 pts.) This problem appears as Problem 2 on the take home part of the exam. You may use this page for rough work.

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Instructor: ANSWERS

Math 10170, Practice Exam I
February 22, 2016

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PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(•)
2.	(a)	(b)	(c)	(•)	(e)
3.	(a)	(•)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(•)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
8.	________
9.	________
10.	________
11.	________
12.	________
Total	________